Published by Institute of Physics Publishing for SISSA/ISAS

Received: *September 27, 2004*
Accepted: *October 22, 2004*

Circular and folded multi-spin strings in spin chain sigma models

Shijong Ryang

Department of Physics, Kyoto Prefectural University of Medicine
Taishogun, Kyoto 603-8334 Japan
E-mail: ryang@koto.kpu-m.ac.jp

Abstract: From the SU(2) spin chain sigma model at the one-loop and two-loop orders we recover the classical circular string solution with two S^5 spins (J_1, J_2) in the $AdS_5 \times S^5$ string theory. In the SL(2) sector of the one-loop spin chain sigma model we explicitly construct a solution which corresponds to the folded string solution with one AdS_5 spin S and one S^5 spin J. In the one-loop general sigma model we demonstrate that there exists a solution which reproduces the energy of the circular constant-radii string solution with three spins (S_1, S_2, J).

Keywords: AdS-CFT and dS-CFT Correspondence, Penrose limit and pp-wave background, Bosonic Strings.

JHEP10(2004)059

Contents

1. Introduction

The AdS/CFT correspondence [1] has more and more revealed the deep relations between the conformal gauge theory and the string theory in curved spaces. The solvability of the string theory in the pp-wave background [2, 3] has opened a new celebrated step to give an interesting proposal identifying nearly point-like string states with gauge invariant near-BPS operators with large R-charge in the BMN limit for the $\mathcal{N} = 4$ SU(N) super Yang-Mills (SYM) theory [4]. Various semiclassical extended string configurations with several large angular momenta in $AdS_5 \times S^5$ which usually go beyond the BMN scaling have been constructed extensively to study the AdS/CFT correspondence for non-BPS states [5]–[13] and reviewed in [14].

In the planar $\mathcal{N} = 4$ SYM theory there has been an important observation that the dilatation operator for the SO(6) sector can be interpreted as a hamiltonian of an integrable spin chain in the one-loop approximation [15, 16]. The planar integrability has been studied at higher loops [17, 18]. It plays important roles for resolving the complicated mixing problem and diagonalizing the dilatation operator in order to obtain the anomalous dimensions of the gauge invariant conformal composite operators by using the powerful method of the Bethe ansatz. Within a closed SU(2) sector consisting of operators of two out of the three complex scalar fields, the one-loop Bethe ansatz of [15] has been extended to three loops in [19] using the three-loop integrability of [17]. An all-loop Bethe ansatz has been proposed and refined in [20].

The Bethe equation for the one-loop dilatation operator in the SU(2) sector has been solved for the states that are dual to the folded and circular strings having two large angular momenta in S^5 [21, 22] to show that its solutions match with the one-loop semiclassical predictions in [9, 11]. At the one-loop and two-loop orders there has been a general proof of the equivalence between the solutions of the Bethe equations in the thermodynamical limit and the classical solutions of the string theory for large conserved charges in the SU(2) sector [23]. Attempts to relate the higher conserved charges on both sides to each other have been made in [24, 25, 23], indicating that the integrable structures on both sides of the correspondence are closely related. Further important works on the “spinning limit” of AdS/CFT have been provided in [26], and various investigations of the gauge/string duality have been presented in [27]–[29].

There arises a natural question about the more direct relation of the integrable spin chain system to the string theory. The SU(2) (Heisenberg $\mathrm{XXX}_{1/2}$) spin chain with the hamiltonian given by the one-loop dilatation operator, in the limit of large number of chain sites, has been described by an effective two-dimensional ferromagnetic sigma model for a coherent-state expectation value of the spin operator, which on the other hand precisely agrees with the sigma model obtained from the rotating string with two spins in S^5 by taking some adequate angular momentum limit in the $AdS_5 \times S^5$ string action [30]. It has been shown that this one-loop identification at the level of actions extends to two loops within the SU(2) sector [31]. The derivation of the low energy effective sigma model from the spin chain system to leading order requires considering only long wavelength spin configurations, and beyond one loop involves quantum corrections from short wavelength modes. From the string theory the same effective sigma model has been derived by choosing a special gauge with a non-diagonal world-sheet metric and implementing an order by order redefinition of the relevant field to get an action linear in the time derivative. The derivation of the SU(2) continuum spin chain sigma model interpolating between the SU(2) ferromagnetic spin chain and the $AdS_5 \times S^5$ string theory has been extended to the SU(3) sector (with three spins in S^5) [32, 33], and the SL(2) sector (with one spin in AdS_5 and one spin in S^5) [33], where a general effective sigma model action is further derived by rearranging the $AdS_5 \times S^5$ string action in the large S^5 spin limit, which generalizes the one in [30, 31] from the two-spin (J_1, J_2) configuration to the more general configuration with two AdS_5 spins and three S^5 spins. There has been a construction of a spin chain sigma model interpolating between the string theory on $AdS_5 \times S^5/\mathbb{Z}_M$ and $\mathcal{N} = 0, 1, 2$ orbifold field theories originating from $\mathcal{N} = 4$ SYM [34].

The one-loop spin chain sigma model for the SU(2) sector has been solved to present a solution which reproduces the first-order correction in the expansion of the classical energy for the folded string solution with two spins in S^5 or the corresponding one-loop term in the anomalous dimension found by using the Bethe ansatz [30]. This solution has been generalized [35] to be related to a general ansatz for rotating strings with two spins which can be reduced to the Neumann-Rosochatius integrable system [13]. The two-loop part of the spin chain sigma model has been also shown to reproduce precisely the second-order energy correction for the folded string solution [31]. The equations of motion for the one-loop spin chain sigma model in the SU(3) sector have been explicitly solved [32] to recover the first-order energy correction for the circular, rational string solution rotating along three orthogonal directions of S^5, which is the constant-radii solution of the Neumann-Rosochatius integrable system [13]. The other solution has been presented [36] and shown to reproduce the circular, elliptic three-spin (J_1, J_2, J_3) string solution [12, 37]. There has been a construction of a general effective one-loop sigma model action with 8-dimensional target space which agrees with a limit of the $AdS_5 \times S^5$ phase-space string action, from which the pulsating solution [8] and its generalization have been reproduced [38].

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We will extend the effective sigma model analysis to other classes of configurations to get a better understanding of the mapping between the classical string theory and the quantum gauge theory. In the continuum SU(2) spin chain sigma model including the first two- "one-loop" and "two-loop"- terms, we will search for a solution which corresponds to the circular string solution with two spins (J_1, J_2) in S^5. We will construct a solution for the SL(2) sector of the one-loop effective sigma model and show how it reproduces the two-spin (S, J) folded string solution in the $AdS_5 \times S^5$ string theory. Manipulating the general effective sigma model action we will present a solution which corresponds to the circular constant-radii solution with two spins in AdS_5 and one spin in S^5.

2. The circular two-spin (J_1, J_2) string solution

The low-energy effective action of the SU(2) ferromagnetic spin chain with the hamiltonian given by the sum of the one-loop [30] and two-loop [31] dilatation operators of the $\mathcal{N} = 4$ SYM theory was given by the expression of the perturbative expansion in the effective coupling constant $\tilde{\lambda} = \lambda/J^2 = g_{YM}^2 N/J^2$

$$S = S_{WZ} - J \int \mathrm{dt} \int_0^{2\pi} \frac{d\sigma}{2\pi} \left\{ \frac{1}{8} \left(\partial_\sigma n^i\right)^2 - \frac{\tilde{\lambda}}{32} \left[\left(\partial_\sigma^2 n^i\right)^2 - \frac{3}{4} (\partial_\sigma n^i)^4 \right] + O(\partial_\sigma^6 n^i) \right\}, \quad (2.1)$$

where S_{WZ} is the Wess-Zumino term and J is the length of the spin chain. The subleading $(\partial_\sigma^2 n^i)^2$ term is given by a naive continuum limit of the expectation value of the two-loop dilatation operator in a coherent state, while the $(\partial_\sigma n^i)^4$ term is derived by taking the continuum limit of the quantum correction to the discrete spin chain theory. The coherent state in the SU(2) ferromagnetic spin chain is specified by a unit vector

$$n_i = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta) \, . \quad (2.2)$$

Alternatively the same effective two-dimensional sigma model action was derived by a suitable limit of the string action in the $R \times S^3 \subset AdS_5 \times S^5$ space-time with metric

$$\begin{aligned} ds^2 &= -dt^2 + |dX_1|^2 + |dX_2|^2 = -dt^2 + d\psi^2 + \cos^2\psi d\varphi_1^2 + \sin^2\psi d\varphi_2^2 \, , \\ X_1 &\equiv X_1 + iX_2 = \cos\psi e^{i\varphi_1} \, , \qquad X_2 \equiv X_3 + iX_4 = \sin\psi e^{i\varphi_2} \, , \end{aligned} \quad (2.3)$$

where the range of the angles is $0 < \psi \leq 2\pi$, $0 < \varphi_1 \leq \pi$, $0 < \varphi_2 \leq \pi$. The string states rotate in the two orthogonal planes (X_1, X_2) and (X_3, X_4) with two angular momenta (J_1, J_2). The three-sphere is parametrized in terms of CP^1 coordinates $U_r (r = 1, 2)$ and a U(1) angle α as $X_1 = U_1 e^{i\alpha}, X_2 = U_2 e^{i\alpha}$ with $U_1 = \cos\psi e^{i\beta}, U_2 = \sin\psi e^{-i\beta}, \alpha = (\varphi_1 + \varphi_2)/2, \beta = (\varphi_1 - \varphi_2)/2$. The common phase α represents a simultaneous rotation in the two plane so that it corresponds to the total spin $J = J_1 + J_2 \equiv \sqrt{\lambda}\mathcal{T}$. Introducing a unit vector n_i belonging to a two-sphere instead of U_r as $n_i \equiv U^\dagger \sigma_i U$, $U = (U_1, U_2)$ where σ_i are Pauli matrices, we have

$$n_i = (\sin 2\psi \cos 2\beta, \sin 2\psi \sin 2\beta, \cos 2\psi) \, , \quad (2.4)$$

$$(ds^2)_{S^3} = (D\alpha)^2 + \frac{1}{4} dn_i^2 = (d\alpha + C)^2 + d\psi^2 + \sin^2 2\psi d\beta^2 \, , \quad C = \cos 2\psi d\beta \quad (2.5)$$

and the string action is expressed in terms of t, α, n_i. The "longitudinal" coordinates t and α are so eliminated through the constraints that the resulting sigma model action is described by a "transverse" coordinate n_i in the same form as (2.1). The leading term quadratic in derivatives in (2.1) is derived in the usual conformal gauge, while in order to obtain the subleading term quartic in derivatives we use a non-diagonal uniform gauge and make a systematic field redefinition order by order in $1/\mathcal{T}$ for elimination of time derivatives [31]. We rescale the time coordinate t as $\mathrm{t} \to \tilde{\lambda}\mathrm{t}$ in the effective action (2.1) and use the coordinate $x = J\sigma/2\pi$ to have

$$S = S_{WZ} - \int d\mathrm{t} \int_0^J dx \left\{ \frac{\lambda}{32\pi^2}(\partial_x n^i)^2 - \frac{\lambda^2}{512\pi^4}\left[(\partial_x^2 n^i)^2 - \frac{3}{4}(\partial_x n^i)^4 \right] \right\}. \tag{2.6}$$

Now from the sigma model action (2.6) we are ready to construct a solution which corresponds to a circular string configuration with two spins J_1 and J_2. At lowest order the equations of motion for the first and second terms in (2.6) give a solution

$$\phi = \omega \mathrm{t}\,, \qquad \theta = \theta(x) \tag{2.7}$$

with

$$\partial_x \theta = \sqrt{a_0 + b_0 \cos\theta}\,, \tag{2.8}$$

where θ and ϕ are polar coordinates for the unit vector n_i in (2.2). The integration of (2.8) is expressed as

$$x = \int_0^\theta \frac{d\theta}{\sqrt{a_0 + b_0\cos\theta}}\,, \tag{2.9}$$

where the integration constant is chosen as $\theta = 0$ at $x = 0$. Here we are interested in the $a_0 > |b_0|$ case which corresponds to the circular two-spin configuration, and perform the integration

$$\sin\frac{\theta(x)}{2} = \mathrm{sn}\left(\frac{\sqrt{a_0+b_0}}{2}x, k \right), \qquad k = \sqrt{\frac{2b_0}{a_0+b_0}}\,. \tag{2.10}$$

By comparing (2.2) with (2.4) we note that $\theta = 2\psi$ and then the range of θ is $0 < \theta \le 4\pi$. Owing to the fundamental period $4K(y_0)$ of sn where $K(y_0)$ is the complete elliptic integral of the first kind, we extract a relation

$$\frac{\sqrt{a_0+b_0}}{2}J = 4K(y_0)\,, \qquad y_0 = k^2 = \frac{2b_0}{a_0+b_0} \tag{2.11}$$

from (2.10) with $\theta(J) = 4\pi$ and $\mathrm{sn}(4K, k) = 0$, where the string wraps completely around a great circle. Alternatively the total angular momentum or the length of the spin chain is also obtained from (2.9) as

$$J = J_1 + J_2 = 4\int_0^\pi \frac{d\theta}{\sqrt{a_0 + b_0\cos\theta}} = \frac{8K(y_0)}{\sqrt{a_0+b_0}}\,. \tag{2.12}$$

The total component of the spin in the z direction is expressed as

$$\begin{aligned} S_3 &= \frac{J_2 - J_1}{2} = -\frac{1}{2}\int_0^J dx \cos\theta = -2\int_0^\pi d\theta \frac{\cos\theta}{\sqrt{a_0+b_0\cos\theta}} \\ &= -\frac{4[(a_0+b_0)E(y_0) - a_0K(y_0)]}{b_0\sqrt{a_0+b_0}}\,, \end{aligned} \tag{2.13}$$

where $E(y_0)$ is the complete elliptic integral of the second kind. From(2.6) the energy of this solution is provided by

$$\begin{aligned} E_1 &= \frac{\lambda}{32\pi^2}\int_0^J dx\{(\partial_x\theta)^2 + \sin^2\theta(\partial_x\phi)^2\} = \frac{\lambda}{32\pi^2}4\int_0^\pi d\theta\frac{a_0 + b_0\cos\theta}{\sqrt{a_0 + b_0\cos\theta}} \\ &= \frac{\lambda}{4\pi^2}\sqrt{a_0 + b_0}E(y_0)\,. \end{aligned} \tag{2.14}$$

Combining (2.14) with (2.12) we have

$$E_1 = \frac{2\lambda}{\pi^2 J}K(y_0)E(y_0) \tag{2.15}$$

and the expression (2.13) with (2.12) gives

$$\frac{J_2}{J} = \frac{1}{y_0} - \frac{E(y_0)}{y_0 K(y_0)}\,. \tag{2.16}$$

In ref. [22] by the semiclassical string approach the circular two-spin string solution in $AdS_5 \times S^5$ was found and characterized by a system of two transcendental equations

$$\left(\frac{\mathcal{E}}{K(y)}\right)^2 - \left(\frac{y\mathcal{T}_1}{E(y) - (1-y)K(y)}\right)^2 = \frac{4}{\pi^2}\,, \tag{2.17}$$

$$\left(\frac{y\mathcal{T}_2}{K(y) - E(y)}\right)^2 - \left(\frac{y\mathcal{T}_1}{E(y) - (1-y)K(y)}\right)^2 = \frac{4}{\pi^2}y\,, \tag{2.18}$$

where $\mathcal{E} = E/\sqrt{\lambda}$, $\mathcal{T}_1 = J_1/\sqrt{\lambda}$, $\mathcal{T}_2 = J_2/\sqrt{\lambda}$. These equations were solved by using the expansions of $y = y_0 + y_1/\mathcal{T}^2 + y_2/\mathcal{T}^4 + \cdots$ and $\mathcal{E} = \mathcal{T} + \epsilon_1/\mathcal{T} + \epsilon_2/\mathcal{T}^3 + \cdots$, i.e. $E = J + \epsilon_1\lambda/J + \epsilon_2\lambda^2/J^3 + \cdots$, and the following solution was presented:

$$\epsilon_1 = \frac{2}{\pi^2}K(y_0)E(y_0)\,, \qquad \frac{J_2}{J} = \frac{1}{y_0} - \frac{E(y_0)}{y_0 K(y_0)}\,. \tag{2.19}$$

The energy expression (2.15) obtained from the one-loop effective hamiltonian of the spin chain sigma model agrees with the leading energy orrection $\epsilon_1\lambda/J$ and the result (2.16) is the same as the momentum fraction J_2/J in (2.19).

Following the prescription in ref. [31], we estimate the subleading energy correction. We perturb the leading-order solution (2.7) with (2.8) keeping J and S_3 fixed so that the energy correction is provided by the evaluation of the subleading term quartic in derivatives in (2.6) on the leading-order solution. The subleading energy correction given by

$$E_2 = -\frac{\lambda^2}{512\pi^4}\int_0^J dx\left[(\partial_x^2\theta)^2 + \frac{1}{4}(\partial_x\theta)^4\right] \tag{2.20}$$

is evaluated on the unperturbed leading-order solution as

$$\begin{aligned} E_2 &= -\frac{\lambda^2}{512\pi^4}4\int_0^\pi \frac{d\theta}{\partial_x\theta}\left[(\partial_x^2\theta)^2 + \frac{1}{4}(\partial_x\theta)^4\right] \\ &= -\frac{\lambda^2}{512\pi^4}\int_0^\pi d\theta\frac{a_0^2 + b_0^2 + 2a_0b_0\cos\theta}{\sqrt{a_0 + b_0\cos\theta}} \\ &= \frac{2}{\pi^4}\frac{\lambda^2}{J^3}K^3(y_0)\{(1-y_0)K(y_0) - (2-y_0)E(y_0)\}\,. \end{aligned} \tag{2.21}$$

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The eq. (2.18) is written by

$$\left(\frac{\mathcal{T}_2/\mathcal{T}}{K(y)-E(y)}\right)^2-\left(\frac{\mathcal{T}_1/\mathcal{T}}{E(y)-(1-y)K(y)}\right)^2=\frac{4}{\pi^2 y}\frac{1}{\mathcal{T}^2}\,, \tag{2.22}$$

which can be expanded in $1/\mathcal{T}^2$ by using the formulae

$$\begin{aligned} K'(y_0) &= \frac{E_0-(1-y_0)K_0}{2y_0(1-y_0)}\,, & K''(y_0) &= \frac{K_0}{4y_0(1-y_0)}-\frac{1-2y_0}{2y_0^2(1-y_0)^2}(E_0-(1-y_0)K_0)\,,\\ E'(y_0) &= -\frac{K_0-E_0}{2y_0}\,, & E''(y_0) &= \frac{K_0-E_0}{4y_0^2}-\frac{E_0-(1-y_0)K_0}{4y_0^2(1-y_0)}\,, \end{aligned} \tag{2.23}$$

where $K_0\equiv K(y_0), E_0\equiv E(y_0)$. The leading term in the expansion gives the momentum fraction J_2/J in (2.19) and the subleading terms of order $1/\mathcal{T}^2$ and order $1/\mathcal{T}^4$ respectively yield

$$y_1 = -\frac{4y_0(1-y_0)K_0^2(K_0-E_0)(E_0-(1-y_0)K_0)}{(E_0^2-(1-y_0)K_0^2)\pi^2}\,, \tag{2.24}$$

$$y_2 = \frac{(1-y_0)(K_0-E_0)(E_0-(1-y_0)K_0)}{E_0^2-(1-y_0)K_0^2}\left(y_1^2A+\frac{4y_1K_0^2}{\pi^2}\right) \tag{2.25}$$

with

$$\begin{aligned} A =& \frac{2}{E_0-(1-y_0)K_0}\left(\frac{y_0K_0''}{2}+K_0'\right)-\frac{y_0K_0(K_0''-E_0'')}{(K_0-E_0)(E_0-(1-y_0)K_0)}+\\ &+\frac{3}{4}\left\{\frac{1}{(1-y_0)^2}\left(\frac{E_0}{K_0-E_0}\right)^2-\left(\frac{K_0}{E_0-(1-y_0)K_0}\right)^2\right\}. \end{aligned} \tag{2.26}$$

To make the expansion of the eq. (2.17) multiplied by $1/\mathcal{T}^2$ we use (2.24), (2.25) and (2.19) to see that the $1/\mathcal{T}^4$ term gives

$$\epsilon_2=\frac{2K_0^3}{\pi^4}[(1-y_0)K_0-(2-y_0)E_0]\,. \tag{2.27}$$

In ref. [19] the two transcendental equations (2.17) and (2.18) for the circular string were rewritten in Lagrange-inversion form, from which the same solution as (2.27) was derived. Thus we have observed that the subleading energy correction E_2 in (2.21) precisely agrees with the second-order energy correction $\epsilon_2\lambda^2/J^3$ in the classical circular string solution with two S^5 spins (J_1, J_2).

3. The folded two-spin (S, J) string solution

Let us consider the general string configuration with two AdS_5 spins (S_1, S_2) and one S^5 spin J. From the classical string action in $AdS_5\times S^5$ the effective two-dimensional sigma model action was constructed [33]. The relevant metric is expressed as

$$ds^2=dY_i^*dY^i+d\alpha^2\,, \tag{3.1}$$

where α is an angle in S^5 and the 3 complex coordinates Y_i $(i = 0, 1, 2)$ parametrize AdS_5 as

$$Y_0 = \cosh\rho e^{it}\,, \quad Y_1 = \sinh\rho\cos\theta e^{i\phi_1}\,, \qquad Y_2 = \sinh\rho\sin\theta e^{i\phi_2} \tag{3.2}$$

and $Y^i = \eta^{ij}Y_j$ with $\eta^{ij} = \mathrm{diag}(-1, 1, 1)$ and $Y_iY^i = -1$. The overall phase y is introduced by $Y_i = e^{iy}V_i$ where V_i are the coordinates of a non-compact version of CP^2. By changing the coodinates as $u \equiv \alpha, v \equiv y - \alpha$ and choosing the conformal gauge supplemented by the condition $u = \alpha = \mathcal{T}\tau$ which fixes the residual conformal diffeomorphism freedom, we have the following leading-order effective sigma model action

$$\begin{aligned} S &= J\int d\mathrm{t}\int_0^{2\pi}\frac{d\sigma}{2\pi}\left\{-(B_{\mathrm{t}} + \partial_{\mathrm{t}}v) - \frac{1}{2}D_\sigma^* V_i^* D_\sigma V^i\right\}, \\ B_a &\equiv iV_i^*\partial_a V^i\ (a = \mathrm{t}, \sigma)\,, \qquad D_\sigma V_i = \partial_\sigma V_i - iB_\sigma V_i\,, \end{aligned} \tag{3.3}$$

where the time coordinate has been rescaled as $\tau = \mathcal{T}\mathrm{t}$. The first term B_{t} plays the role of a Wess-Zumino term and here we leave the total derivative term $\partial_{\mathrm{t}}v$. The "longitudinal" coordinate v is redundant but plays a role when it comes to the conformal gauge constraints.

Now we focus on the two-spin SL(2) sector with one AdS_5 spin $(S_1 = S, S_2 = 0)$ and one S^5 spin J, whose relevant metric is given by

$$ds^2 = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\phi_1^2 + d\alpha^2\,. \tag{3.4}$$

We choose

$$t = y + \eta\,, \qquad \phi_1 = y - \eta, \qquad \theta = 0 \tag{3.5}$$

to have $V_i = (\cosh\rho e^{i\eta}, \sinh\rho e^{-i\eta}, 0)$ and

$$ds^2 = -(dy + B)^2 + d\alpha^2 + d\rho^2 + \sinh^2 2\rho d\eta^2\,, \qquad B = \cosh 2\rho d\eta\,, \tag{3.6}$$

from which y is regarded as a time coordinate of the target space. The leading-order effective sigma model action (3.3) becomes

$$S = J\int d\mathrm{t}\int_0^{2\pi}\frac{d\sigma}{2\pi}\left\{-\cosh 2\rho\partial_{\mathrm{t}}\eta - \frac{1}{2}[(\partial_\sigma\rho)^2 + \sinh^2 2\rho(\partial_\sigma\eta)^2]\right\}, \tag{3.7}$$

where the total derivative term is here dropped. This sigma model action was also produced from the SL(2) coherent state expectation value of the hamiltonian of the integrable $\mathrm{XXX}_{-1/2}$ spin chain in the gauge theory side [33]. By rescaling the time coordinate $\mathrm{t} \to \tilde{\lambda}\mathrm{t}$ and using the space coordinate $x = J\sigma/2\pi$, we have

$$S = \int d\mathrm{t}\int_0^J dx\left\{-\cosh 2\rho\partial_{\mathrm{t}}\eta - \frac{\lambda}{8\pi^2}[(\partial_x\rho)^2 + \sinh^2 2\rho(\partial_x\eta)^2]\right\}. \tag{3.8}$$

The equations of motion read

$$-2\sinh 2\rho\partial_{\mathrm{t}}\eta + \frac{\lambda}{4\pi^2}[\partial_x^2\rho - 2\sinh 2\rho\cosh 2\rho(\partial_x\eta)^2] = 0\,, \tag{3.9}$$

$$-2\sinh 2\rho\partial_{\mathrm{t}}\rho + \frac{\lambda}{4\pi^2}\partial_x(\sinh^2 2\rho\partial_x\eta) = 0 \tag{3.10}$$

with the boundary conditions

$$\eta(x = J, \mathrm{t}) = \eta(x = 0, \mathrm{t}) \,, \qquad \rho(x = J, \mathrm{t}) = \rho(x = 0, \mathrm{t}) \,. \tag{3.11}$$

We start to make an ansatz $\partial_x \eta = 0$ obeying (3.11). The eq. (3.10) yields $\partial_{\mathrm{t}} \rho = 0$. Then the derivative of (3.9) with respect to t gives $\partial_{\mathrm{t}}^2 \eta = 0$, from which we have $\partial_{\mathrm{t}} \eta = \omega_\eta$. The eq. (3.9) turns out to be

$$\partial_x^2 \rho - \frac{8\pi^2 \omega_\eta}{\lambda} \sinh 2\rho = 0 \,, \tag{3.12}$$

whose first integral is given by $(\partial_x \rho)^2/2 - (4\pi^2 \omega_\eta/\lambda) \cosh 2\rho = c$. Thus we obtain

$$(\partial_x \rho)^2 = a - b \cosh 2\rho, \qquad a = 2c \,, \qquad b = -\frac{8\pi^2 \omega_\eta}{\lambda} \,. \tag{3.13}$$

Here we consider the $\omega_\eta < 0$ case which corresponds to $b > 0$ as well as $c > 0$, that is $a > 0$. When $a > b$, the folded string configuration is formed such that the maximal value of ρ is determined by $a - b \cosh 2\rho_0 = 0$.

Therefore the conserved momentum P_η conjugate to η is specified by

$$P_\eta = -\int_0^J dx \cosh 2\rho = -4 \int_0^{\rho_0} d\rho \frac{1 + 2\sinh^2 \rho}{\sqrt{a - b \cosh 2\rho}} \,, \tag{3.14}$$

while the angular momentum associated with $u \equiv \alpha$ or the length of the spin chain is expressed as

$$J = \int_0^J dx = 4 \int_0^{\rho_0} d\rho \frac{1}{\sqrt{a - b \cosh 2\rho}} \,. \tag{3.15}$$

The integrations can be performed in terms of complete elliptic integrals as

$$\begin{aligned} P_\eta &= -\frac{4}{\sqrt{2b}} (2E(x) - K(x)) = -\frac{8}{\sqrt{2b}} (E(x) - K(x)) - J \,, \\ J &= \frac{4}{\sqrt{2b}} K(x) \,, \qquad x = -\sinh^2 \rho_0 = -\frac{a - b}{2b} \,. \end{aligned} \tag{3.16}$$

Since v is interpreted as a time coordinate and $i\partial_v = i\partial_y$, the one-loop energy provided from (3.8) is expressed as

$$E_v^{(1)} = \frac{\lambda}{8\pi^2} \int_0^J dx [(\partial_x \rho)^2 + \sinh^2 2\rho (\partial_x \eta)^2] \tag{3.17}$$

and the energy in the coordinates of (3.6) is given by

$$E_y = E_v = J + E_v^{(1)} \,. \tag{3.18}$$

When the solution is substituted into (3.17), we have

$$E_v^{(1)} = \frac{\lambda}{2\pi^2} \int_0^{\rho_0} d\rho \frac{a - b \cosh 2\rho}{\sqrt{a - b \cosh 2\rho}} = \frac{\lambda}{8\pi^2} (aJ + bP_\eta) \,. \tag{3.19}$$

The conformal dimension of the gauge invariant operator is identified with the energy in the global coordinates in AdS_5 whose relevant metric is here described by (3.4), so that the AdS_5 energy is given by $E_t = i\partial_t$ and the AdS_5 spin by $S = -i\partial_{\phi_1}$. From the coordinate

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transformation in (3.5) we find the relations between the relevant conserved charges

$$E_y = i\partial_y = i(\partial_t + \partial_{\phi_1}) = E_t - S\,, \tag{3.20}$$

$$-P_\eta = i\partial_\eta = i(\partial_t - \partial_{\phi_1}) = E_t + S\,. \tag{3.21}$$

Combining (3.20) with (3.18), (3.19) and (3.16) we obtain

$$E_t = J + S + \frac{2\lambda}{\pi^2}\frac{K(x)}{J}[(1-x)K(x) - E(x)]\,. \tag{3.22}$$

The substitution of (3.22) and (3.16) into (3.21) gives to the leading order

$$1 + \frac{S}{J} = \frac{E(x)}{K(x)}\,. \tag{3.23}$$

The obtained expressions (3.22) and (3.23) agree with the results in ref. [22] where the folded two-spin (S, J) string solution in $AdS_5 \times S^5$ is shown to be related by an analytic continuation to the folded two-spin (J_1, J_2) string solution and the system of two transcendental equations specifying this configuration is solved by the expansion procedure.

4. The circular three-spin (S_1, S_2, J) string solution

Let us turn to the three-spin (S_1, S_2, J) string configuration in the relevant $AdS_5 \times S^5$ metric

$$ds^2 = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho(d\theta^2 + \cos^2\theta d\phi_1^2 + \sin^2\theta d\phi_2^2) + d\alpha^2\,. \tag{4.1}$$

Setting

$$t = y + \eta\,, \qquad \phi_1 = y - \eta\,, \qquad \phi_2 = y + \phi \tag{4.2}$$

we have

$$\begin{aligned} V_i &= \left(\cosh\rho e^{i\eta}, \sinh\rho\cos\theta e^{-i\eta}, \sinh\rho\sin\theta e^{i\phi}\right), \\ B_{\mathrm{t}} &= \left(\cosh^2\rho + \sinh^2\rho\cos^2\theta\right)\partial_{\mathrm{t}}\eta - \sinh^2\rho\sin^2\theta\partial_{\mathrm{t}}\phi\,. \end{aligned} \tag{4.3}$$

Through the rescaling of the time coordinate $\mathrm{t} \to \tilde{\lambda}\mathrm{t}$ the effective sigma model action (3.3) is expressed in terms of σ here as

$$\begin{aligned} S = &-\frac{J}{2\pi}\int d\mathrm{t}\int_0^{2\pi} d\sigma\left\{(\cosh^2\rho + \sinh^2\rho\cos^2\theta)\dot{\eta} - \sinh^2\rho\sin^2\theta\dot{\phi} + \dot{v}\right\} - \\ &-\frac{\lambda}{4\pi J}\int d\mathrm{t}\int_0^{2\pi} d\sigma\Big\{\rho'^2 - (\cosh^2\rho - \sinh^2\rho\cos^2\theta)\eta'^2 + \sinh^2\rho\theta'^2 + \sinh^2\rho\sin^2\theta\phi'^2 + \\ &\qquad + \left[(\cosh^2\rho + \sinh^2\rho\cos^2\theta)\eta' - \sinh^2\rho\sin^2\theta\phi'\right]^2\Big\}\,, \end{aligned} \tag{4.4}$$

where the dot and prime denote derivatives with respect to t and σ. This action is invariant under constant shifts in η and ϕ so that the corresponding conserved angular momenta are given by

$$P_\eta = -\frac{J}{2\pi}\int_0^{2\pi} d\sigma(\cosh^2\rho + \sinh^2\rho\cos^2\theta)\,, \tag{4.5}$$

$$P_\phi = \frac{J}{2\pi}\int_0^{2\pi} d\sigma \sinh^2\rho\sin^2\theta\,. \tag{4.6}$$

The effective hamiltonian is written by

$$H = \frac{\lambda}{4\pi J}\int_0^{2\pi} d\sigma\Big\{\rho'^2 - (\cosh^2\rho - \sinh^2\rho\cos^2\theta)\eta'^2 + \sinh^2\rho\theta'^2 + \sinh^2\rho\sin^2\theta\phi'^2 + \\ + [(\cosh^2\rho + \sinh^2\rho\cos^2\theta)\eta' - \sinh^2\rho\sin^2\theta\phi']^2\Big\}\,. \tag{4.7}$$

From the action (4.4) the equations of motion for η, ϕ, ρ, θ are obtained respectively by

$$\partial_t(\cosh^2\rho + \sinh^2\rho\cos^2\theta) - \frac{\lambda}{J^2}\{\partial_\sigma[(\cosh^2\rho - \sinh^2\rho\cos^2\theta)\eta'] - \\ - \partial_\sigma[(\cosh^2\rho + \sinh^2\rho\cos^2\theta)\times \\ \times((\cosh^2\rho + \sinh^2\rho\cos^2\theta)\eta' - \\ - \sinh^2\rho\sin^2\theta\phi')]\} = 0\,, \tag{4.8}$$

$$\partial_t(\sinh^2\rho\sin^2\theta) + \frac{\lambda}{J^2}\{-\partial_\sigma(\sinh^2\rho\sin^2\theta) + \\ + \partial_\sigma[\sinh^2\rho\sin^2\theta((\cosh^2\rho + \sinh^2\rho\cos^2\theta)\eta' - \\ \sinh^2\rho\sin^2\theta\phi')]\} = 0\,, \tag{4.9}$$

$$(1+\cos^2\theta)\dot\eta - \sin^2\theta\dot\phi + \frac{\lambda}{2J^2}\Big\{-\frac{\rho''}{\sinh\rho\cosh\rho} - \sin^2\theta\eta'^2 + \theta'^2 + \sin^2\theta\phi'^2 + \\ + 2[(\cosh^2\rho + \sinh^2\rho\cos^2\theta)\eta' - \sinh^2\rho\sin^2\theta\phi']\times \\ \times\big((1+\cos^2\theta)\eta' - \sin^2\theta\phi'\big)\Big\} = 0\,, \tag{4.10}$$

$$\sinh^2\rho(\dot\eta + \dot\phi) + \frac{\lambda}{2J^2}\Big\{\frac{\partial_\sigma(\sinh^2\rho\theta')}{\sin\theta\cos\theta} + \sinh^2\rho\eta'^2 - \sinh^2\rho\phi'^2 + \\ + 2[(\cosh^2\rho + \sinh^2\rho\cos^2\theta)\eta' - \sinh^2\rho\sin^2\theta\phi']\times \\ \times\sinh^2\rho(\eta' + \phi')\Big\} = 0\,. \tag{4.11}$$

In order to solve the involved equations of motion we make a simple ansatz; $\rho = \rho_0$ and $\theta = \theta_0$ with ρ_0 and θ_0 constant, and $\eta' = k_1, \phi' = k_2, v' = p$ where k_1, k_2, p are integers. Substituting this ansatz into the equations of motion we see that (4.8) and (4.9) are trivially satisfied and (4.11) yields

$$\dot\eta + \dot\phi = \frac{\lambda}{2J^2}\{k_2^2 - k_1^2 - 2[k_1(\cosh^2\rho_0 + \sinh^2\rho_0\cos^2\theta_0) - k_2\sinh^2\rho_0\sin^2\theta_0](k_1+k_2)\}\,. \tag{4.12}$$

It is important to note that the relations between the conserved charges for the three-spin configuration corresponding to (3.20) and (3.21) are given by

$$E_y = i\partial_y = i(\partial_t + \partial_{\phi_1} + \partial_{\phi_2}) = E_t - S_1 - S_2\,, \tag{4.13}$$
$$-P_\eta = i\partial_\eta = i(\partial_t - \partial_{\phi_1}) = E_t + S_1\,, \tag{4.14}$$
$$-P_\phi = i\partial_\phi = i\partial_{\phi_2} = -S_2\,, \tag{4.15}$$

where the coordinate transformation in (4.2) has been taken into account and S_1 and S_2 are the spins coming from the rotations in the ϕ_1 and ϕ_2 directions. The spin S_2 is so identified with P_ϕ in (4.6) that we have

$$S_2 = J\sinh^2\rho_0\sin^2\theta_0\,. \tag{4.16}$$

The eq. (4.13) combines with $E_y = J + E_v^{(1)}$ corresponding to (3.18), and yields

$$E_t = J + S_1 + S_2 + E_v^{(1)}\,, \tag{4.17}$$

where the one-loop energy correction $E_v^{(1)}$ is evaluated from the effective hamiltonian (4.7). Collecting eqs. (4.5), (4.14) and (4.17) we obtain to the leading order

$$2S_1 + S_2 = J\sinh^2\rho_0(1+\cos^2\theta_0)\,, \tag{4.18}$$

which gives $S_1 = J\sinh^2\rho_0\cos^2\theta_0$, that is compared with (4.16). Combining (4.12) with (4.10) substituted by the ansatz we derive

$$\dot\phi = -\frac{\lambda}{2J^2}\left\{k_1^2 - k_2^2 + 2\left[k_1 + \frac{2k_1S_1}{J} + \frac{(k_1-k_2)S_2}{J}\right]k_2\right\}, \tag{4.19}$$
$$\dot\eta = -\frac{\lambda}{J^2}\left[k_1 + \frac{2k_1S_1}{J} + \frac{(k_1-k_2)S_2}{J}\right]k_1\,. \tag{4.20}$$

The one-loop energy correction is also evaluated as

$$E_v^{(1)} = \frac{\lambda}{2J^3}\{-k_1^2J(J+S_2) + k_2^2JS_2 + [k_1(J+2S_1+S_2) - k_2S_2]^2\}\,. \tag{4.21}$$

Since the momentum along the σ-direction should vanish, we find from the conformal gauge constraint that

$$P_\sigma = -\frac{\lambda}{2\pi}\int d\sigma[(\cosh^2\rho + \sinh^2\rho\cos^2\theta)\partial_\sigma\eta - \sinh^2\rho\sin^2\theta\partial_\sigma\phi + \partial_\sigma v] = 0\,, \tag{4.22}$$

which yields

$$2k_1S_1 + (k_1-k_2)S_2 + (p+k_1)J = 0\,. \tag{4.23}$$

If we identify $2k_1 = n_1,\ k_1 - k_2 = n_2,\ p + k_1 = m$ and take account of (4.23), the clumsy expression (4.21) turns out to be a symmetric one

$$E_v^{(1)} = \frac{\lambda}{2J^2}(m^2J + \sum_{a=1}^{2} n_a^2 S_a) \tag{4.24}$$

with $mJ + \sum_{a=1}^2 n_aS_a = 0$. Thus we recover the result of ref. [13], where the general circular constant-radii solution in $AdS_5\times S^5$ is constructed by reducing the $AdS_5\times S^5$ classical string theory to the Neumann-Rosochatius integrable system.

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5. Conclusion

By constructing the explicit solution to the SU(2) spin chain sigma model and reproducing precisely the leading and subleading energy corrections of the circular string solution with two spins (J_1, J_2) in the $AdS_5 \times S^5$ string theory, we have demonstrated the validity of the one-loop [30] and two-loop [31] expressions of the SU(2) spin chain sigma model action.

For the SL(2) spin chain sigma model we have made an ansatz to solve its equations of motion, and shown that this solution reproduces the leading enegy correction of the folded string solution with two spins (S, J). From the general effective sigma model action we have recovered the leading energy correction of the circular constant-radii string solution with three spins (S_1, S_2, J). Its involved equations of motion are solved by a simple ansatz and the obtained energy expression in terms of the three spins is out of order at the first glance. However, by performing an appropriate redefinition of the relevant parameters of the solution and taking account of the conformal gauge constraint, we can extract a simple and symmetric energy expression. Since the energy of the classical string solution in the global coordinates in AdS_5 can be identified with the conformal dimemsion of the composite operator in the gauge theory, we have observed that there appears a subtlety in extracting the relevant energy from the solution of the effective sigma model for both (S, J) and (S_1, S_2, J) sectors, while the (J_1, J_2) sector does not include such subtlety. The subtlety is connected with the choice of the time coordinate of the target space and resolved by making use of the relations between the relevant conserved charges associated with the coordinate transformation. Although there is such difference, the folded (S, J) sector exhibits some resemblance to the folded (J_1, J_2) sector as expected from the analytic continuation arguement.

Thus by making simple ansätze we have constructed several particular classical solutions to the effective sigma models in order to confirm the idea that there should exist the effective two-dimensional sigma model interpolating between the quantum planar SYM theory and the classical string theory in $AdS_5 \times S^5$. It would be interesting to search for the other type of solution by making the different kind of ansatz for the (S_1, S_2, J) sector.

While this paper being written, we learned of the work [39] where the SL(2) spin chain sigma model was constructed by using an approach different from [32] and making a different coordinate transformation, and the folded two-spin (S, J) string solution was reproduced.

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